ECS 455 Chapter 1

Introduction & Review

1.2 Fourier Transform and Communication System

Office Hours:
BKD 3601-7
Wednesday 15:30-16:30
Friday 9:30-10:30

Notes #1

- Fourier Transform
- Modulation
- DSB-SC and QAM

Sirindhorn International Institute of Technology
Thammasat University
School of Information, Computer and Communication Technology

ECS 455: Mobile Communications
Fourier Transform and Communication Systems

Prapun Suksompong, Ph.D.
prapun@siit.tu.ac.th

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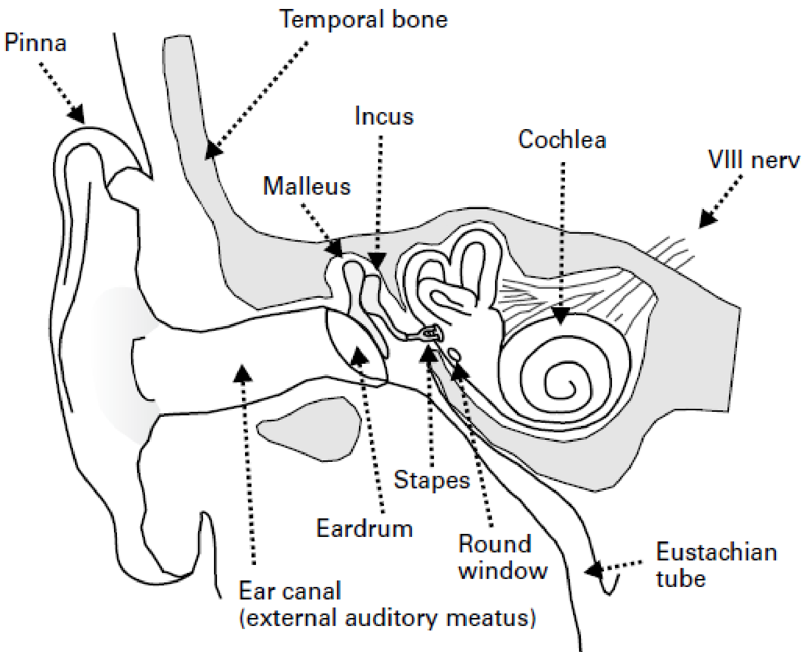
Communication systems are usually viewed and analyzed in frequency domain. This note reviews some basic properties of Fourier transform and introduce basic communication systems.

Contents

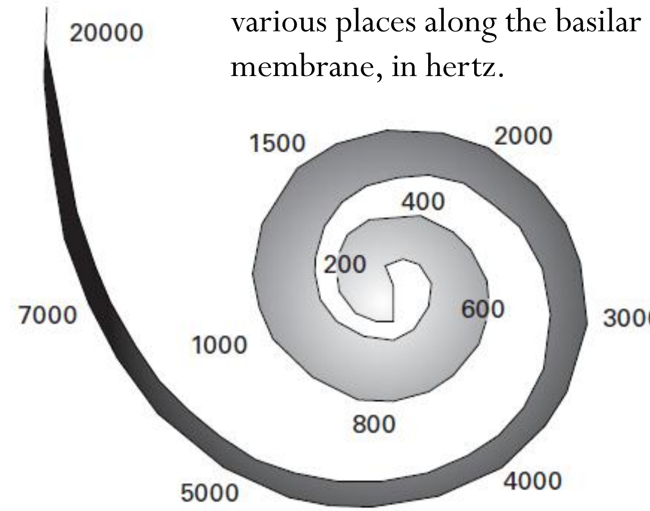
The cochlea has sometimes been described as a **biological Fourier analyzer**.

Fourier Transform in Auditory System



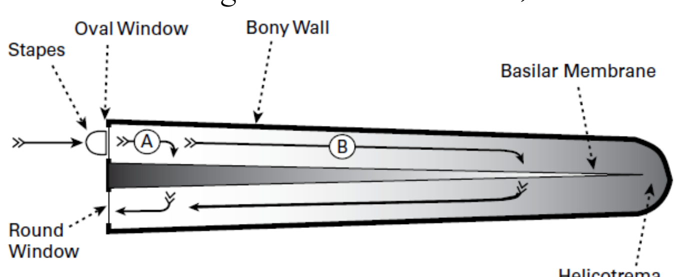
[Schnupp, Nelken, and King, 2010, Fig 2.1]

Approximate best frequencies of various places along the basilar membrane, in hertz.

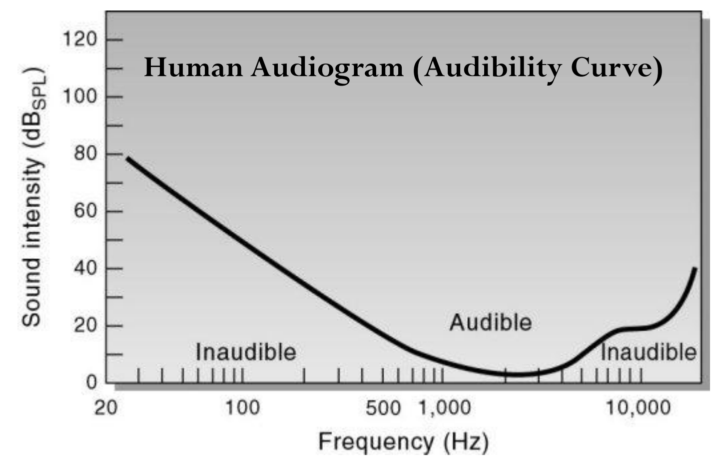


[Schnupp, Nelken, and King, 2010, Fig 2.2]

Schematic showing the cochlea unrolled, in cross-section.



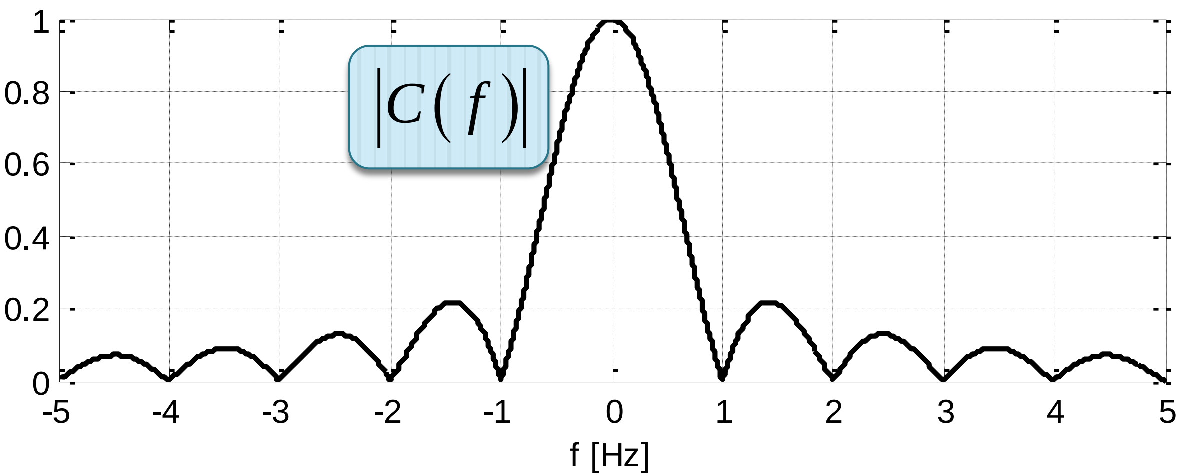
[Schnupp, Nelken, and King, 2010, Fig 2.2]



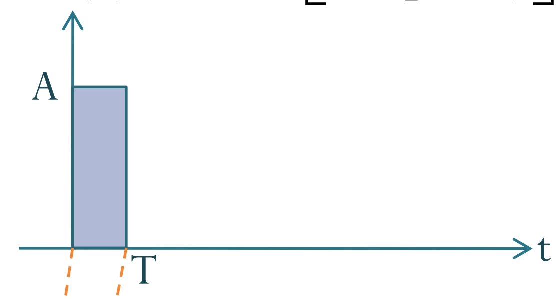
[http://psyc254.uconn.edu/Lecture18/]

Spectrum of Digital Data (1/4)

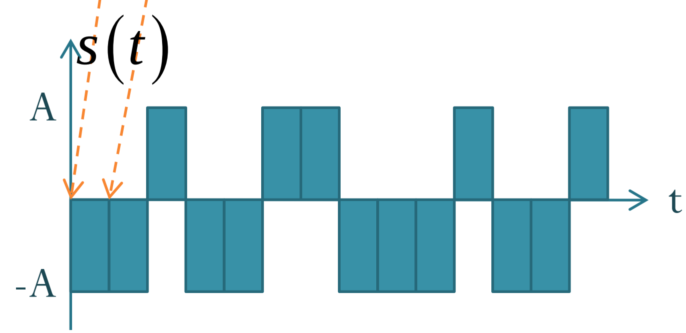
$\left(A=1, T=1\right)$



$$c(t) = A \times 1\left[t \in [0,T)\right]$$



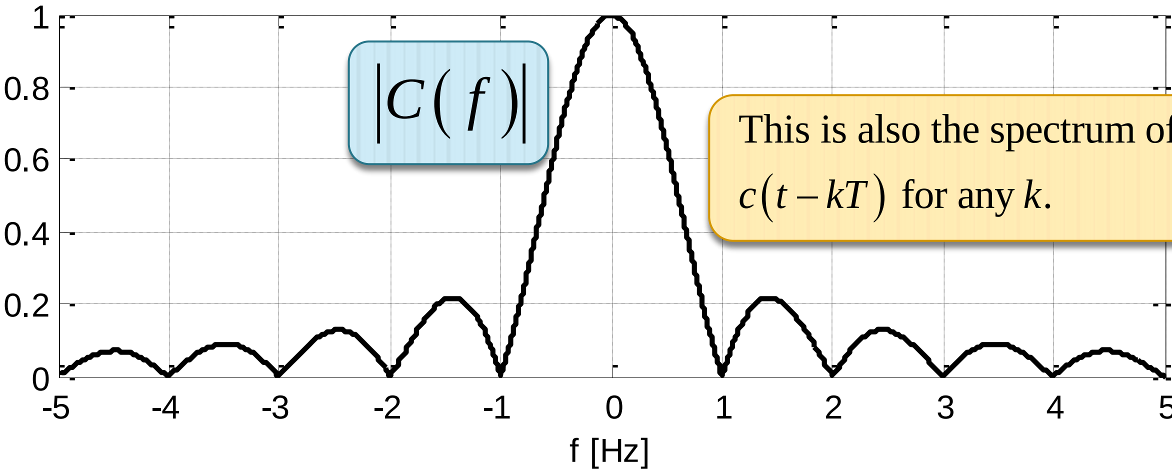
m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



Can you sketch the spectrum of $s(t)$?

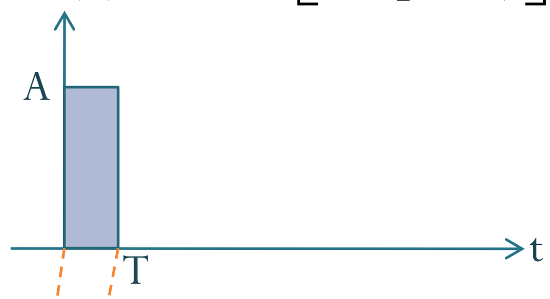
Spectrum of Digital Data (2/4)

$(A=1, T=1)$



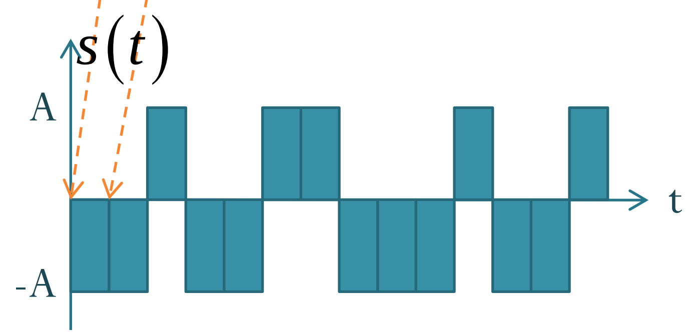
This is also the spectrum of $c(t-kT)$ for any k.

$$c(t) = A \times 1\left[t \in [0,T)\right]$$



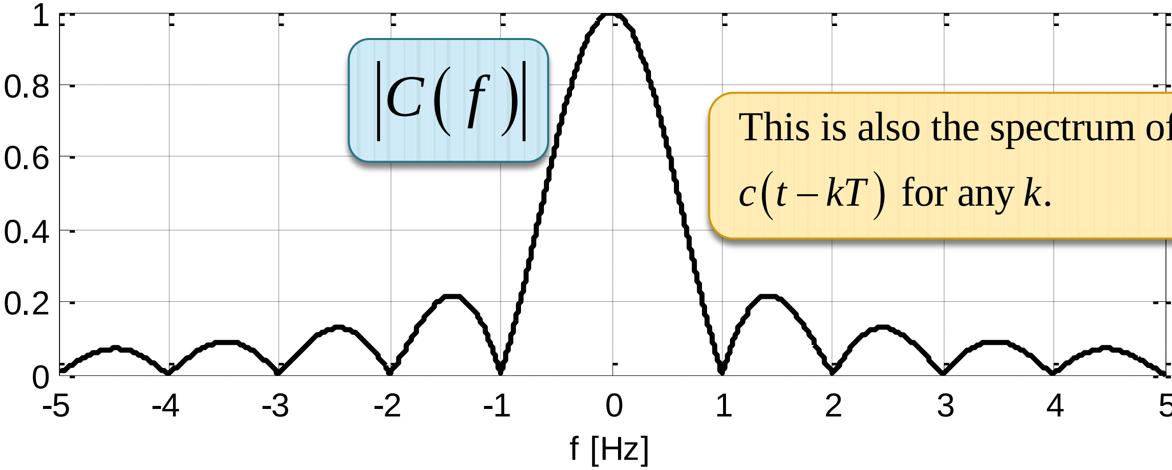
m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

$$s(t) = \sum_{k=0}^{n-1} m_k c(t-kT)$$



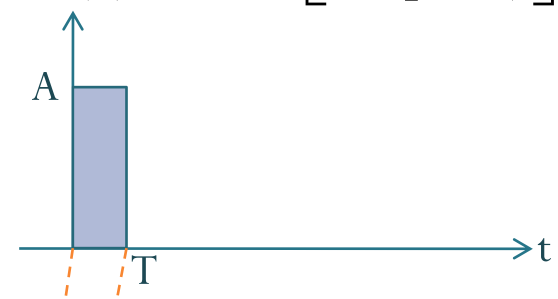
Spectrum of Digital Data (3/4)

$(A=1, T=1)$

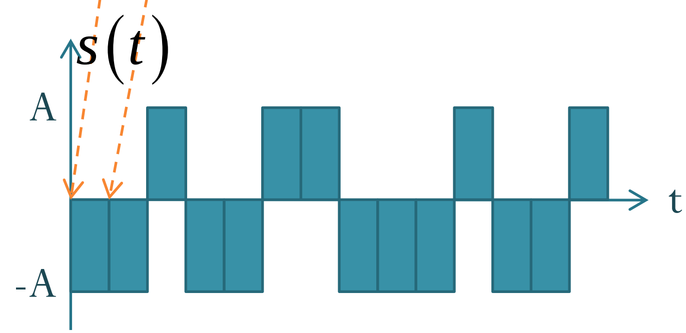


This is also the spectrum of $c(t-kT)$ for any k.

$$c(t) = A \times 1\left[t \in [0, T)\right]$$



m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

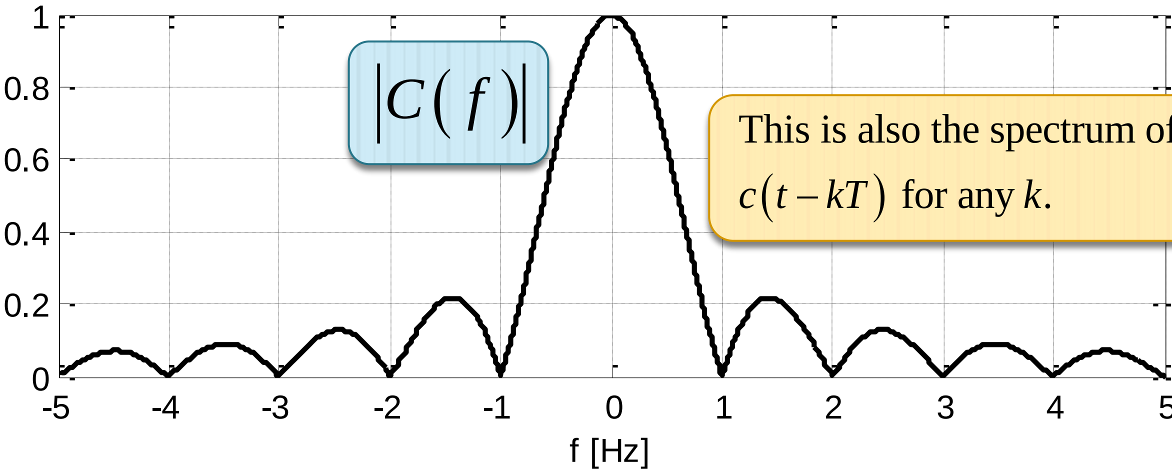


$$s(t) = \sum_{k=0}^{n-1} m_k c(t-kT)$$

$$\xrightarrow{\mathcal{F}} S(f) = C(f) \sum_{k=0}^{n-1} m_k e^{-j2\pi fkT}$$

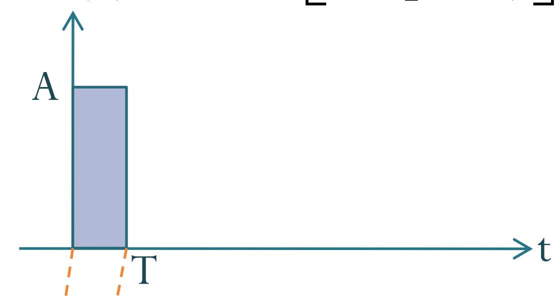
Spectrum of Digital Data (4/4)

$(A=1, T=1)$

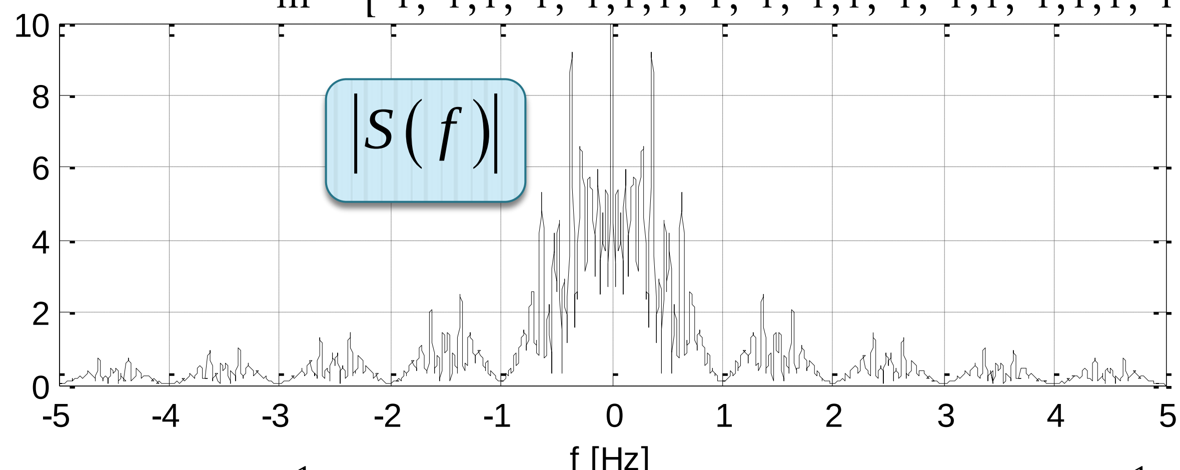


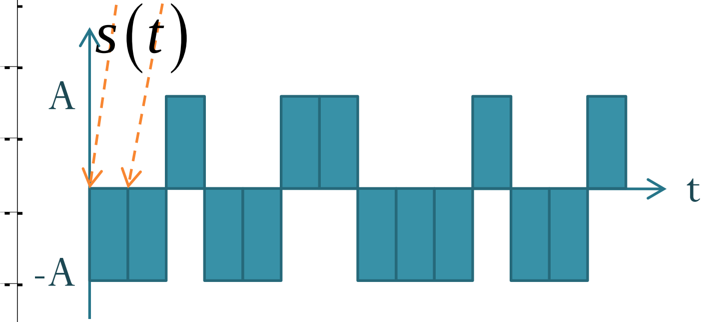
This is also the spectrum of $c(t-kT)$ for any k.

$$c(t) = A \times 1[t \in [0,T)]$$



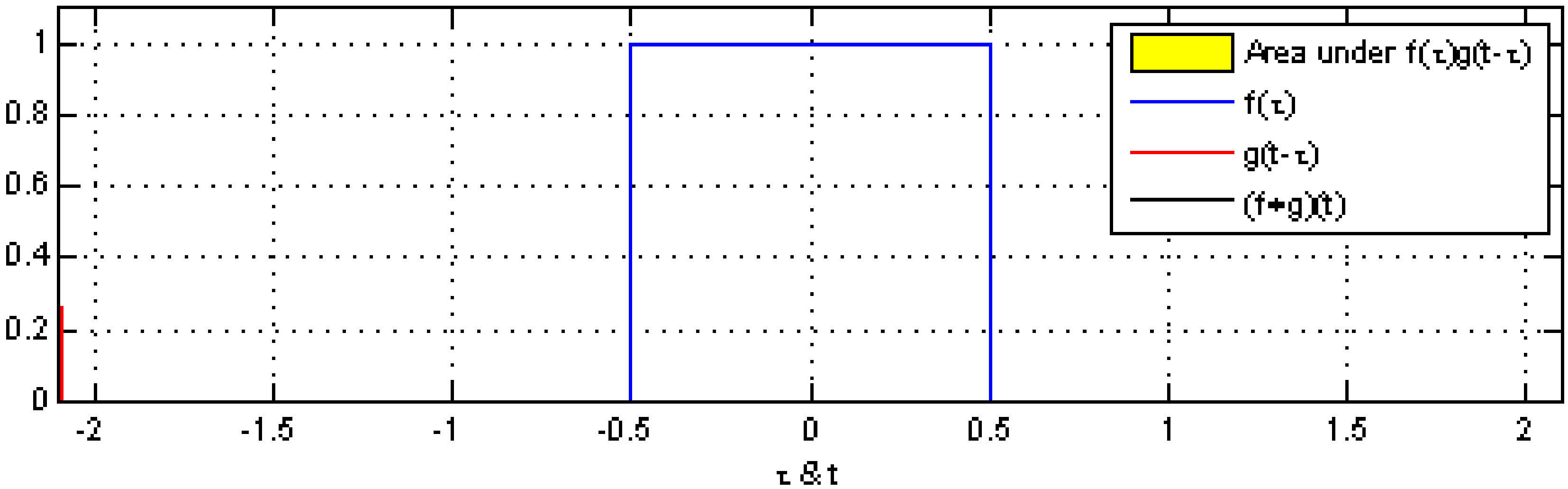
m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



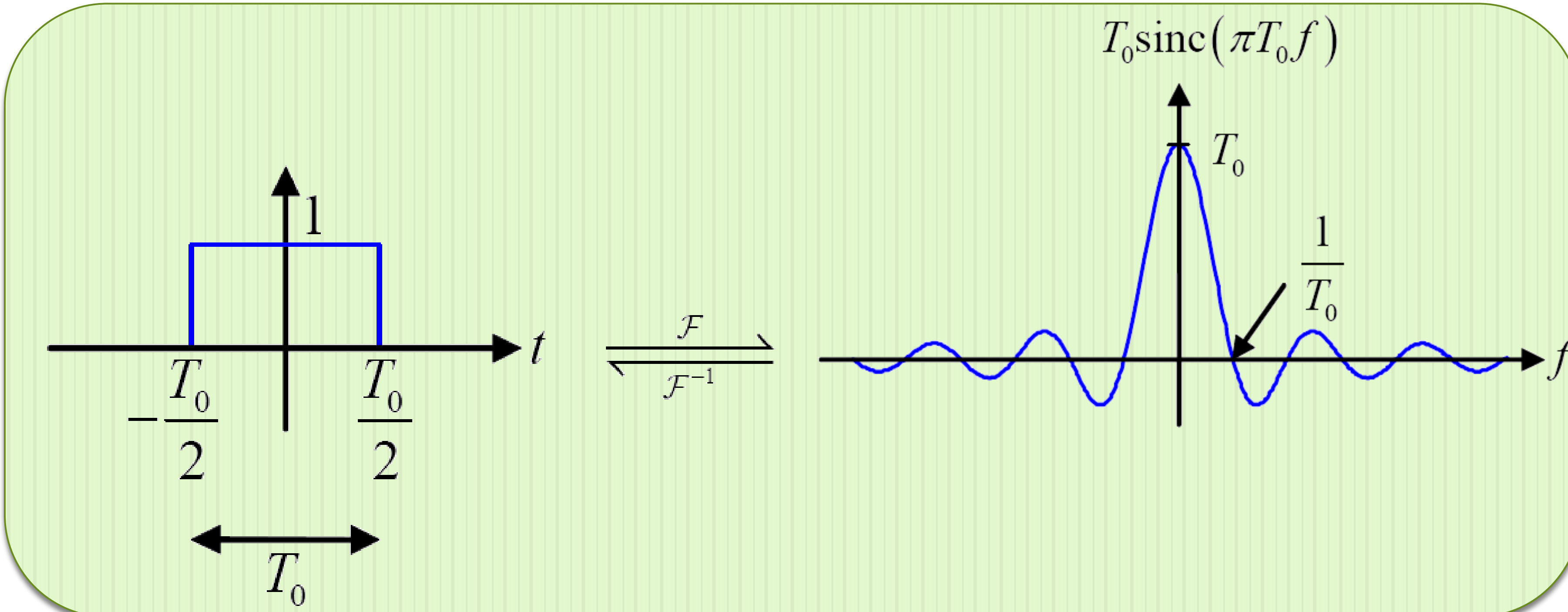


$$s(t) = \sum_{k=0}^{n-1} m_k c(t-kT) \xrightarrow{\mathcal{F}} S(f) = C(f) \sum_{k=0}^{n-1} m_k e^{-j2\pi fkT}$$

Example: Convolution



Frequency-Domain Analysis



Shifting Properties: $g(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} G(f)$ $\quad e^{j2\pi f_0 t} g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f-f_0)$

Modulation: $m(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}M(f-f_c)+\frac{1}{2}M(f+f_c)$

Important Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$2\cos^2 x = 1 + \cos(2x)$$

$$2\sin^2 x = 1 - \cos(2x)$$

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

$$\cos(2\pi f_c t + \theta) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}\delta(f - f_c)e^{j\theta} + \frac{1}{2}\delta(f + f_c)e^{-j\theta}$$

$$g(t - t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} G(f)$$

$$e^{j2\pi f_0 t} g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f - f_0)$$

$$m(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c)$$

Instantaneous Frequency (Ex 1/6)

- Suppose you want the frequency of

$$\cos\left(2\pi ft\right)$$

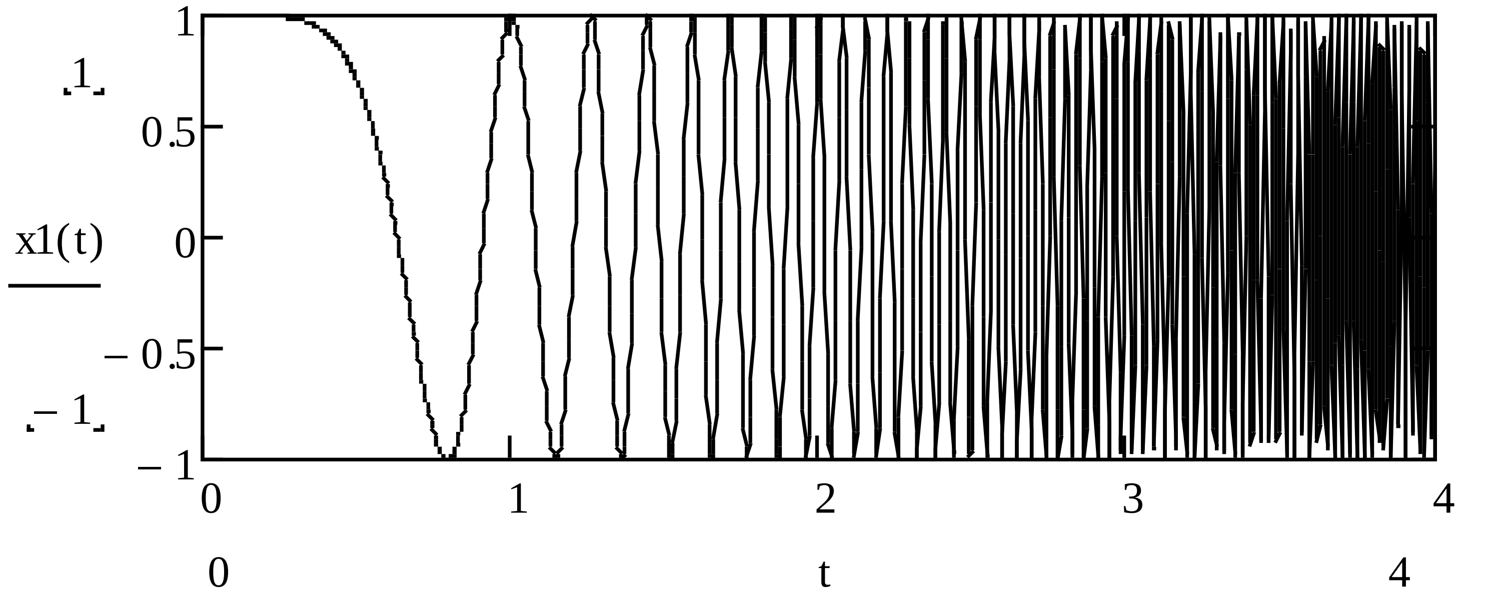
  to change as a function of time $f\left(t\right)=t^2$
- Intuitively, the following substitution makes sense:

$$\cos\left(2\pi\left(t^2\right)t\right)$$

- But will it work?

Instantaneous Frequency (Ex 2/6)

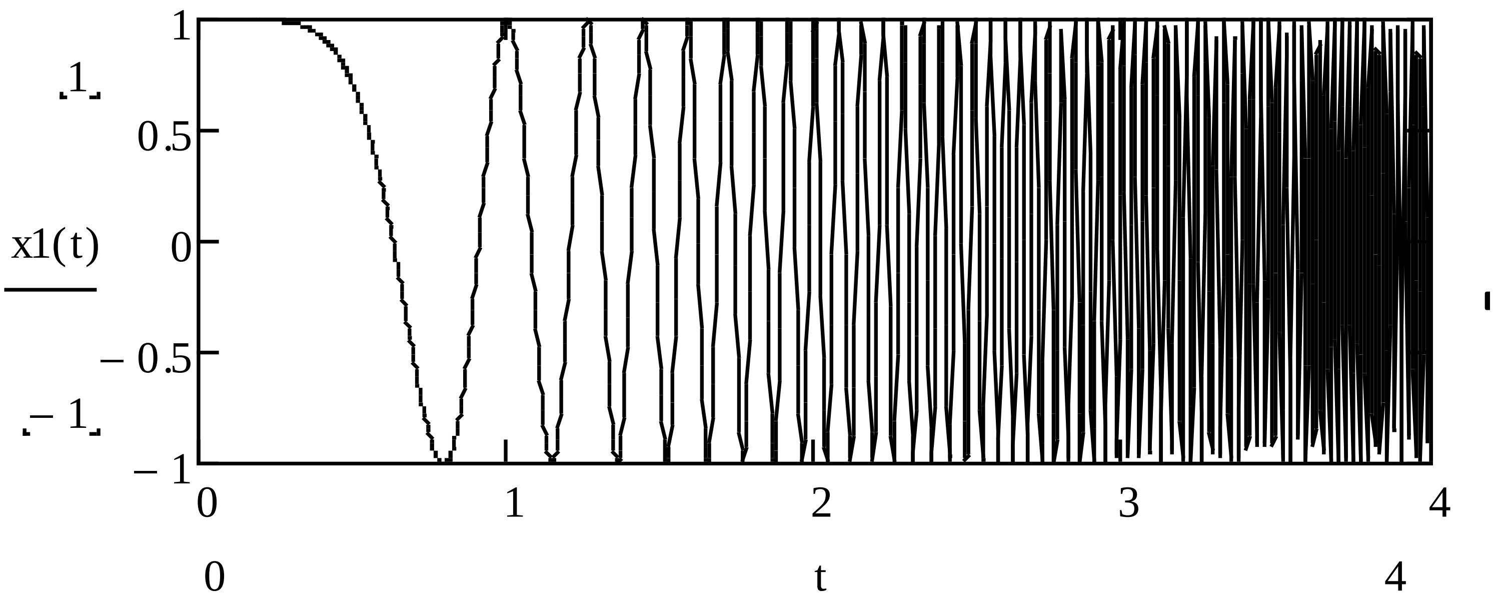
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



At t = 2, frequency = ?

Instantaneous Frequency (Ex 3/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

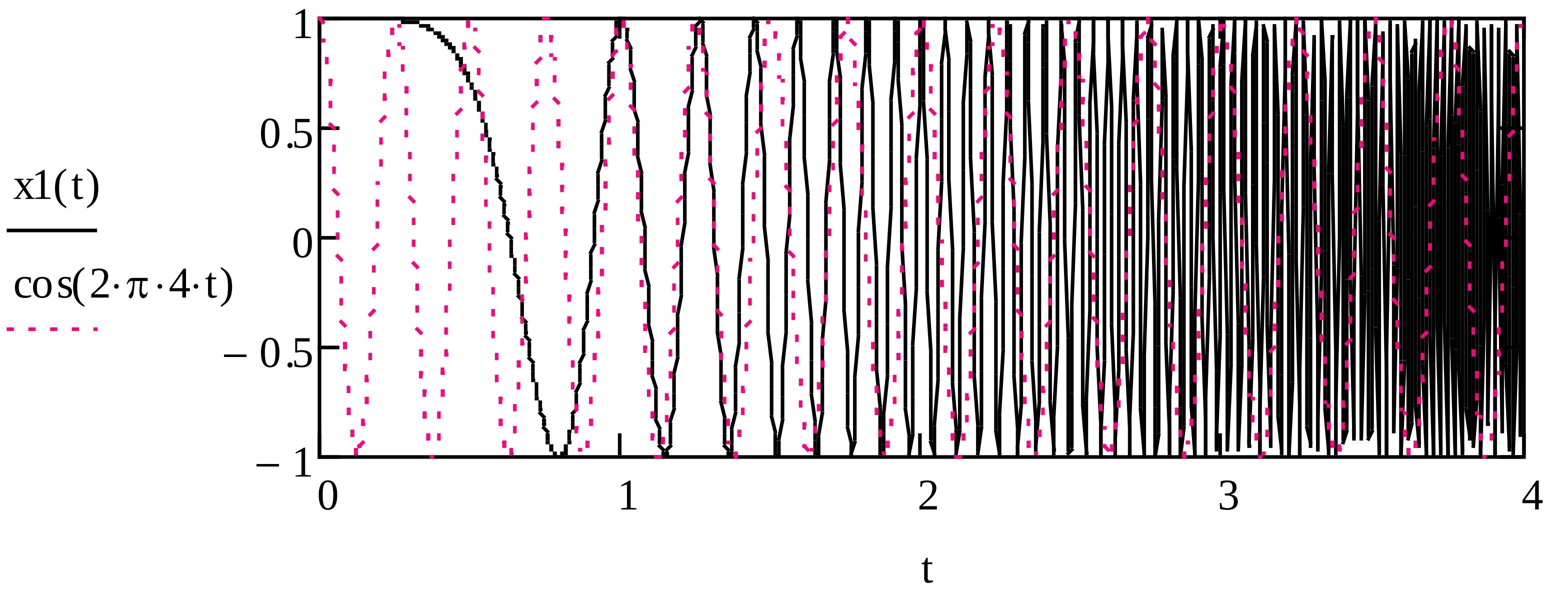


$\cos\left(2\pi ft\right)$ → At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frequency (Ex 4/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

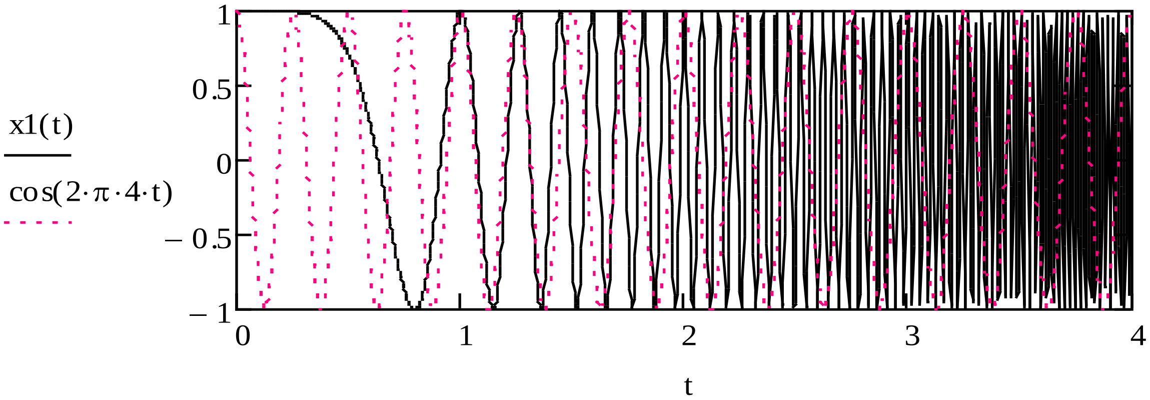


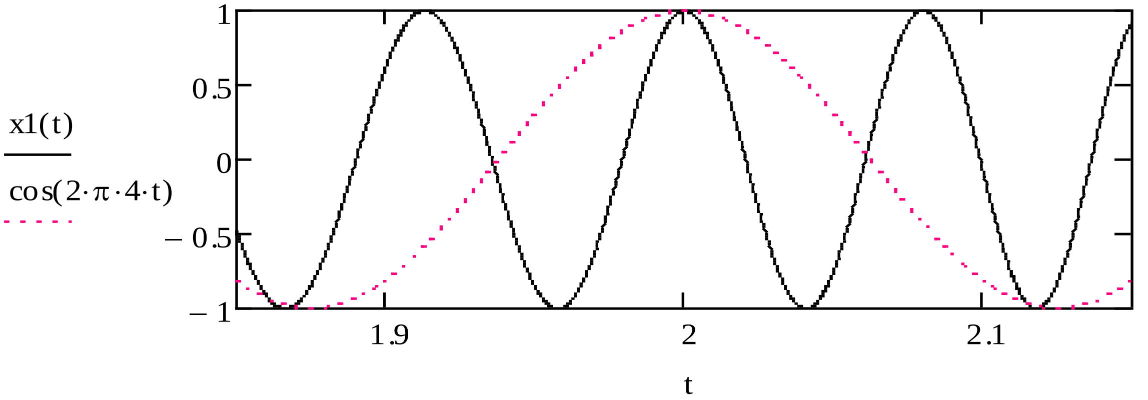
$\cos\left(2\pi f t\right)$ → At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frequency (Ex 5/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

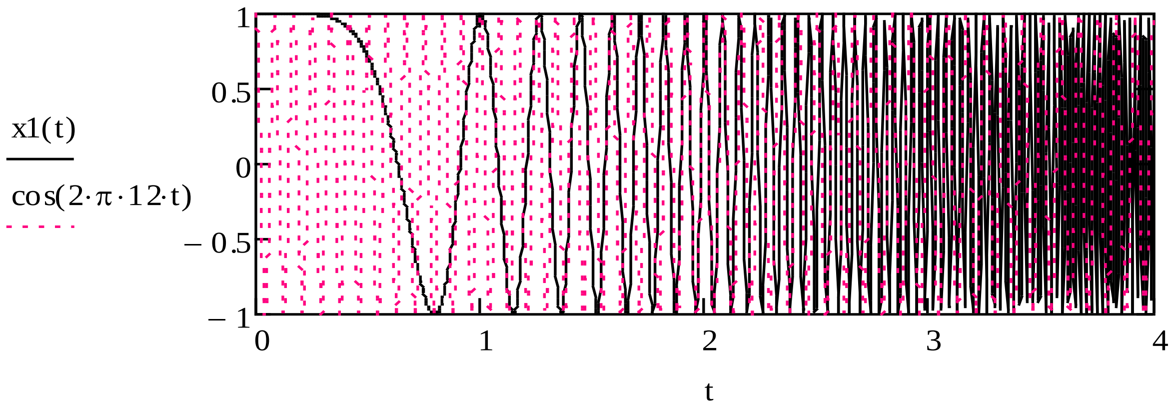


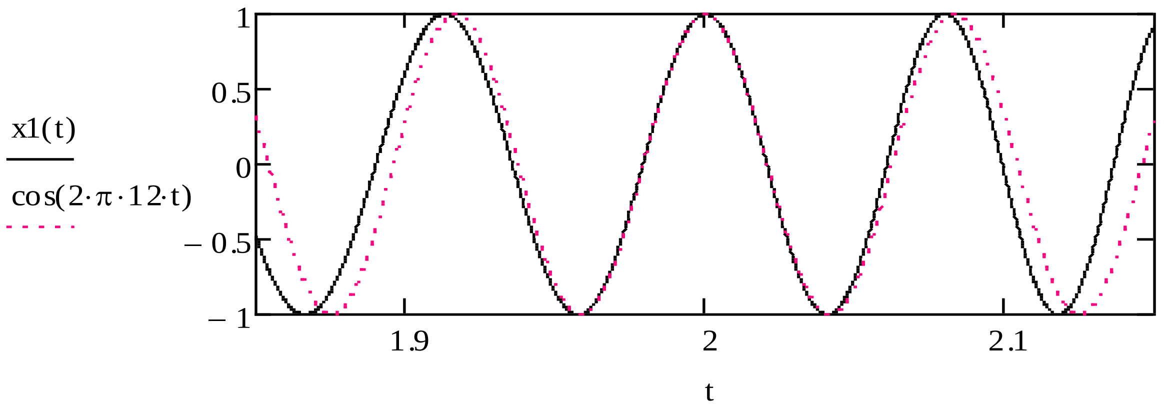


At t = 2, $\cos\left(2\pi\left(t^2\right)t\right)$ oscillates much faster than 4Hz.

Instantaneous Frequency (Ex 6/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$





At t = 2, the frequency of $\cos\left(2\pi\left(t^2\right)t\right)$ is closer to 12 Hz!?

Instantaneous Frequency

of Generalized Sinusoids $x(t) = A\cos(\theta(t))$

$$f(t) = \frac{1}{2\pi}\theta'(t)$$

QAM

$$s(t) = \overbrace{m_I(t)}^{\text{In-phase component}}\cos(\omega_c t) - \overbrace{m_Q(t)}^{\text{Quadrature component}}\sin(\omega_c t)$$

$$= \mathrm{Re}\left\{\underbrace{\left(m_I(t) + jm_Q(t)\right)}_{m(t)} e^{j\omega_c t}\right\}$$

- Complex baseband signal
- Complex envelope of $s(t)$
- Compex lowpass equivalent signal of $s(t)$